

SeaHorn: A CHC-based Verification Tool













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LOPSTR/PPDP, Sep 5, 2018



2007

- [j1]     Mario Méndez-Lojo, Jorge A. Navas, Manuel V. Hermenegildo:
An Efficient, Parametric Fixpoint Algorithm for Analysis of Java Bytecode. Electr. Notes Theor. Comput. Sci. 190(1): 51-60
- [c4]     Jorge A. Navas, Edison Mera, Pedro López-García, Manuel V. Hermenegildo:
User-Definable Resource Bounds Analysis for Logic Programs. ICLP 2007: 348-363
- [c3]     Mario Méndez-Lojo, Jorge A. Navas, Manuel V. Hermenegildo:
A Flexible, (C)LP-Based Approach to the Analysis of Object-Oriented Programs. LOPSTR 2007: 154-168



Automated Reasoning for Software

Model Checking



Abstract Interpretation



Symbolic Execution

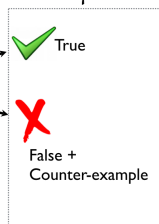


Input



Automated Reasoning

Output



True = code satisfies the safety requirement + certificate

False = code violates the safety requirement + cex



Building Automated Reasoning Tools is Time Consuming

Parse the program

Produce an optimized intermediate representation with a reduced number of cases

Build a verification engine

Support for procedures, pointers, arrays, etc.



Minimize effort when facing a new verification task

build reusable logic-based verification technology and static analysis techniques

Useful to **software developers**:

efficient, user-friendly, trusted, certificate-producing, ...

Useful to **researchers** in verification

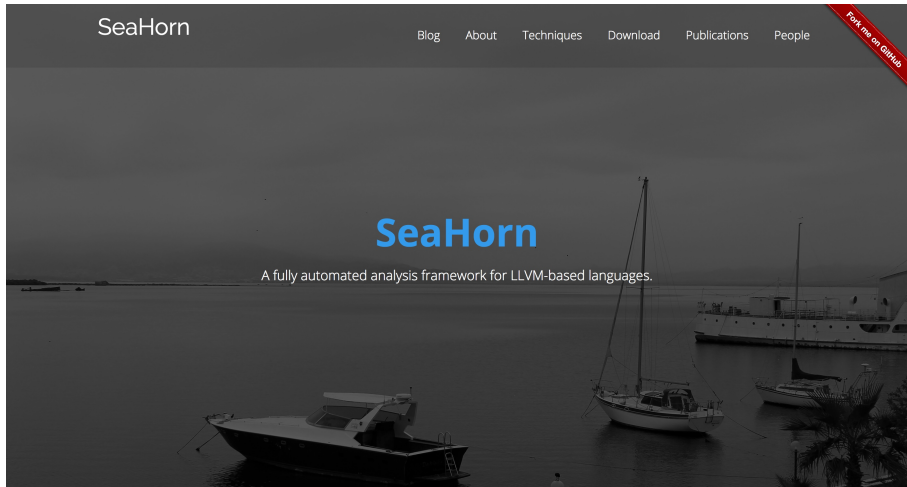
help to assess the effectiveness of a new idea as quick as possible

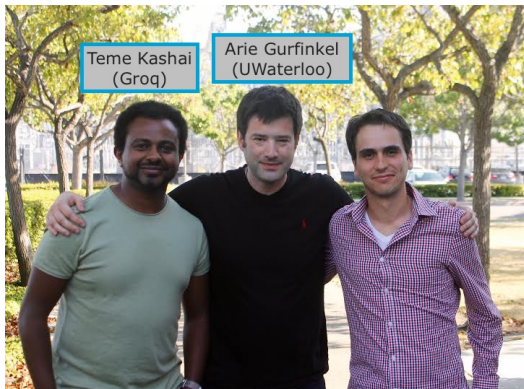


In this talk ...

- 1 SeaHorn Overview
- 2 Demo
- 3 Constrained Horn Clauses for Verification
- 4 Solving CHCs
- 5 Conclusions and Current/Future Work



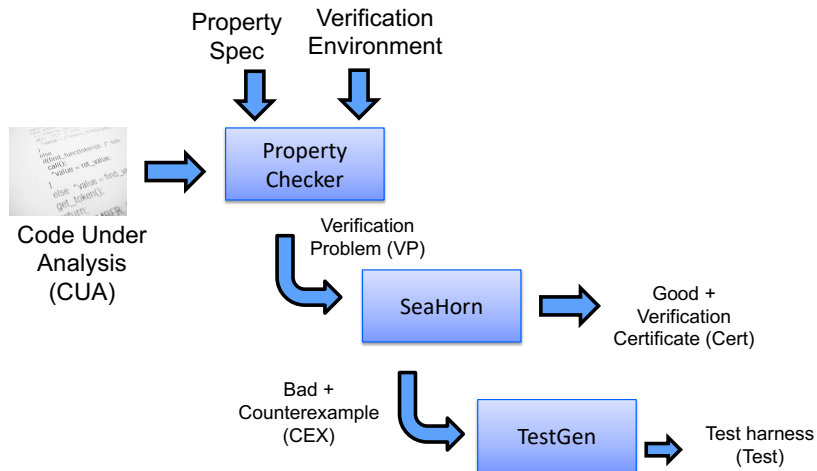




And many great collaborators such as Bjorner, Gange, Komuravelli, Sondergaard, Stuckey, etc



SeaHorn Workflow



Writing a Property Checker

Similar to a dynamic checker (e.g., clang sanitizers)

A significant development effort for each new property

- new specialized static analyses to rule out trivial cases

- different instrumentations have affect on performance

Developed by a domain expert

- understanding of verification techniques is useful (but not required)

- 3-6 month effort for a new property

 - but many things can be reused between similar properties (out-of-bounds, null-deref, taint checking, use-after-free, etc)

SeaHorn property checkers

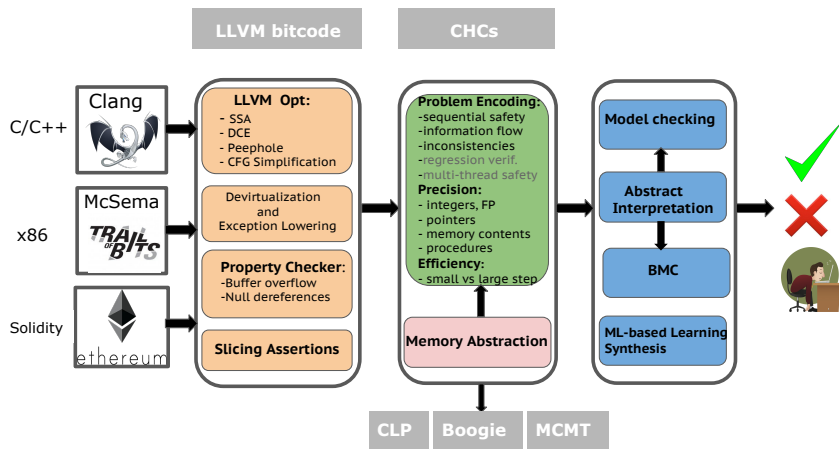
- memory safety (out of bounds, null pointer)

 - ongoing work to improve scalability and usability

- taint analysis (developed by Princeton)



SeaHorn Architecture





- 1 SeaHorn Overview
- 2 Demo
- 3 Constrained Horn Clauses for Verification**
- 4 Solving CHCs
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Constrained Horn Clauses (CHCs)

A Constrained Horn Clause (CHC) is a formula:

$$\forall \mathcal{V} \cdot \left(\underbrace{\phi}_{\text{constraint}} \wedge \underbrace{p_1(X_1) \wedge \dots \wedge p_k(X_k)}_{\text{body}} \rightarrow \underbrace{h(X)}_{\text{head}} \right), \text{ for } k \geq 0$$

non-linear

\mathcal{F} : function symbols, \mathcal{P} : predicate symbols, and \mathcal{V} : variables
 ϕ is a constraint over \mathcal{F} and \mathcal{V} wrt some background theory

$X_i, X \subseteq \mathcal{V}$ are (possibly empty) vectors of variables

$p_i(X_i)$ is an application $p(t_1, \dots, t_n)$ of an n -ary $p \in \mathcal{P}$ for FO terms t_i constructed from \mathcal{F} and X_i

$h(X)$ is either defined analogously to p_i or false



Satisfiability of CHCs

A **model** of a set of CHCs is an interpretation \mathcal{J} of each predicate p_i that makes all clauses valid

A set of CHCs is **satisfiable** if it has a model, and is unsatisfiable otherwise

In the context of verification:

- a program satisfies a property iff its corresponding CHCs are satisfiable

- models for CHCs correspond to inductive invariants and summaries

- derivations to false correspond to counterexample



CHCs are expressive enough to model a broad set of interesting verification and inference problems

CHCs are very amenable for abstractions



$Init(X) \rightarrow Inv(X)$

$Inv(X) \wedge Step(X, X') \rightarrow Inv(X')$

$Inv(X) \rightarrow \neg Bad(X)$



Predicate Abstraction and Refinement for Verifying Multi-Threaded Programs

Ashutosh Gupta Corneliu Popeea Andrey Rybalchenko

$$\begin{aligned} & \bigwedge_{i \in \{1, \dots, N\}} (\text{Init}(X) \rightarrow \text{Inv}_i(X)) \\ & \bigwedge_{i \in \{1, \dots, N\}} (\text{Inv}_i(X) \wedge \text{Step}_i(X, X') \rightarrow \text{Inv}_i(X')) \\ & \bigwedge_{i \in \{1, \dots, N\}} (\text{Inv}_i(X) \wedge \text{Env}_i(X, X') \rightarrow \text{Inv}_i(X')) \\ & \bigwedge_{i, j \in \{1, \dots, N\}, i \neq j} (\text{Inv}_j(X) \wedge \text{Step}_j(X, X') \rightarrow \text{Env}_i(X, X')) \\ & \text{Inv}_1(X) \wedge \dots \wedge \text{Inv}_N(X) \rightarrow \neg \text{Bad}(X) \end{aligned}$$



$$\begin{aligned} & \text{Init}(X, A) \rightarrow \text{Inv}(X, A) \\ & I(X, A) \wedge \text{Step}(X, A, X', A') \rightarrow I(X', A') \\ & \text{Inv}(X, A) \rightarrow \neg \text{Bad}(X, A) \end{aligned}$$

Step can contain array constraints of the form:

$$\begin{aligned} a' &= \text{write}(a, i, v) \\ v &= \text{read}(a, i) \end{aligned}$$

where $i, v \in X \cup X'$, $a \in A$, and $a' \in A'$



Verification of Array Manipulating Programs

Cell morphing: from array programs to array-free Horn clauses*

David Monniaux

Laure Gonnord

Abstract array a into a pair (k, a_k) st. $a[k] = a_k$

$a' = \text{write}(a, i, v)$ (“the value at i is v , the rest unchanged”):

$$i = k \wedge \text{Inv}(X, v, i, a_k) \rightarrow \text{Inv}(X, v, i, v)$$

$$i \neq k \wedge \text{Inv}(X, v, k, a_k) \rightarrow \text{Inv}(X, v, k, a_k)$$

$v = \text{read}(a, i)$ (“ v has new value, the rest is preserved”):

$$i = k \wedge \text{Inv}(X, v, i, a_i) \rightarrow \text{Inv}(X, a_i, i, a_i)$$

$$i \neq k \wedge \underline{\text{Inv}(X, v, k, a_k)} \wedge \text{Inv}(X, v, i, a_i) \rightarrow \text{Inv}(X, a_i, k, a_k)$$



Finding Inconsistencies in Programs with Loops*

Temesghen Kahsai¹, Jorge A. Navas², Dejan Jovanović³, Martin Schäfer³

Verifying Array Programs by Transforming Verification Conditions

Emanuele De Angelis¹, Fabio Fioravanti¹,
Alberto Pettorossi², and Maurizio Proietti³

Automating Regression Verification

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SMT-Based Verification of Parameterized Systems

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Horn Clauses for Communicating Timed Systems

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Cornell University, USA

Philipp Rümmer Pavle Subotic Wang Yi
Uppsala University, Sweden



A Hoare triple $\{Pre\}P\{Post\}$ is valid iff every terminating execution of P that starts in a state satisfying Pre ends in a state satisfying $Post$

Validity of Hoare triples can be reduced to FOL validity by applying a predicate transformer, e.g., the Dijkstra's **weakest liberal precondition**:

$$\{Pre\}P\{Post\} \iff Pre \Rightarrow wlp(P, Post)$$



Translating to CHCs Using Weakest Liberal Preconditions

$$Pre \rightarrow wlp(Main, Post) \wedge \bigwedge_{f \in P} \forall x, r. wlp(B_f, \mathcal{S}_f(x, r))$$

$$\begin{aligned} wlp(\text{if } C \ S_1 \ \text{else } S_2, \phi) &\sim C \rightarrow wlp(S_1, \phi) \wedge \neg C \rightarrow wlp(S_2, \phi) \\ wlp(S_1; S_2, \phi) &\sim wlp(S_1, wlp(S_2, \phi)) \\ wlp(x = e, \phi) &\sim \phi[x \leftarrow e] \\ wlp(\text{error}, \phi) &\sim \perp \\ wlp(\text{while } C \ B, \phi) &\sim \mathcal{I}(\bar{x}) \wedge \\ &\quad \forall \bar{x} ((\mathcal{I}(\bar{x}) \wedge C \wedge \rightarrow wlp(B, \mathcal{I}(\bar{x}))) \wedge \\ &\quad (\mathcal{I}(\bar{x}) \wedge \neg C \rightarrow \phi)) \\ wlp(x = f(y), \phi) &\sim \forall r. \mathcal{S}_f(y, r) \rightarrow \phi[x \leftarrow r] \end{aligned}$$

And apply negation, prenex, and conjunctive normal form

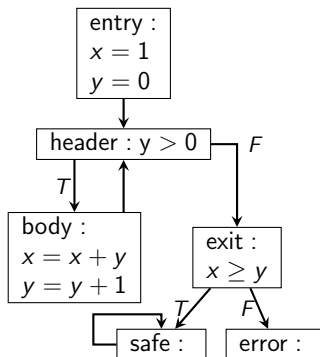


Translating to CHCs Using Dual WLP

```
main() {  
  x = 1;  
  y = 0;  
  while (y > 0) {  
    x = x + y;  
    y = y + 1;  
  }  
  assert(x ≥ y)  
}
```

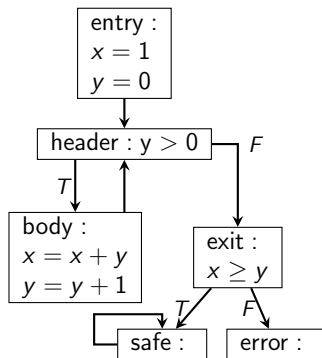


Translating to CHCs Using Dual WLP



Translating to CHCs Using Dual WLP

$$\text{wlp}(P, \text{Post}) = \neg \text{wlp}(P, \neg \text{Post})$$



$\text{entry}(x, y) \leftarrow \text{true}.$

$h(x, y) \leftarrow \text{entry}(x, y), x = 1, y = 0.$

$b(x, y) \leftarrow h(x, y), y > 0.$

$h(x', y') \leftarrow b(x, y),$
 $x' = x + y, y' = y + 1.$

$\text{exit}(x, y) \leftarrow h(x, y), y \leq 0.$

$\text{error}(x, y) \leftarrow \text{exit}(x, y), x < y.$



Rule for `if-then-else` can cause the resulting CHCs to be exponentially larger than the original program

Solution: generate compact VCs for loop-free code

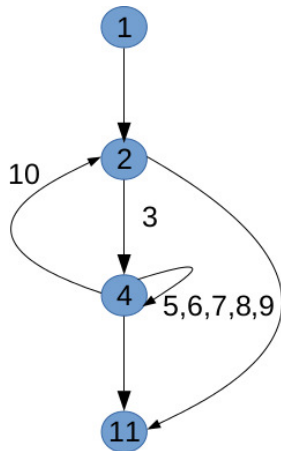
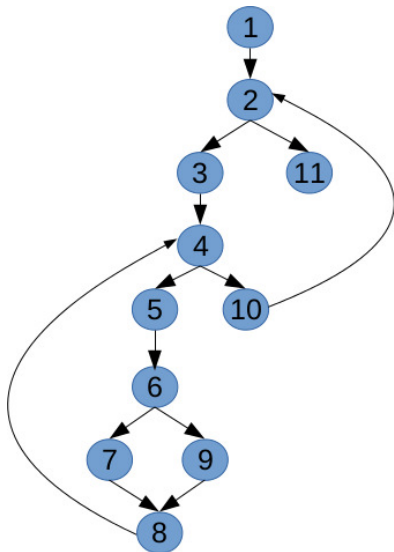
Use of **Cut-point graph (CPG)** rather than the original CFG

A CPG is a summarized CFG, where each node represents a cut-point (loop head) and each edge represents multiple loop-free paths through the CFG

CPGs preserve reachability of control locations



From CFG to CPG



Large-Step Encoding using Cut-point Graphs

Single Static Assignment (SSA): every value has a unique definition

```
int x, y, n;  
l0: x = 0; y = *;  
l1: while (x < n) {  
  l2: if (y > 0)  
    l3: x = x + y;  
    else  
  l4: x = x - y;  
  l5: y = -1 × y;  
}  
l6:
```

```
l0: goto l1  
l1: x0 = φ(0 : l0, x3 : l5);  
    y0 = φ(y : l0, y1 : l5);  
    if (x0 < n) goto l2 else goto l6  
l2: if (y0 > 0) goto l3 else goto l4  
l3: x1 = x0 + y0; goto l5  
l4: x2 = x0 - y0; goto l5  
l5: x3 = φ(x1 : l3, x2 : l4);  
    y1 = -1 × y0  
    goto l1  
l6:
```



Large-Step Encoding using Cut-point Graphs

ϕ :

$x_1 = x_0 + y_0 \wedge$
 $x_2 = x_0 - y_0 \wedge$
 $y_1 = -1 \times y_0 \wedge$
 $B_2 \rightarrow x_0 < n \wedge$
 $B_3 \rightarrow B_2 \wedge y_0 > 0 \wedge$
 $B_4 \rightarrow B_2 \wedge y_0 \leq 0 \wedge$
 $B_5 \rightarrow ((B_3 \wedge x_3 = x_1) \vee$
 $(B_4 \wedge x_3 = x_2)) \wedge$
 $B_5 \wedge x'_0 = x_3 \wedge y'_0 = y_1$

l_0 : ~~goto~~ l_T
 l_1 : $x_0 = \phi(0 : l_0, x_3 : l_5);$
 $y_0 = \phi(y : l_0, y_1 : l_5);$
if $(x_0 < n)$ **goto** l_2 **else goto** l_6
 l_2 : **if** $(y_0 > 0)$ **goto** l_3 **else goto** l_4
 l_3 : $x_1 = x_0 + y_0$; **goto** l_5
 l_4 : $x_2 = x_0 - y_0$; **goto** l_5
 l_5 : $x_3 = \phi(x_1 : l_3, x_2 : l_4);$
 $y_1 = -1 \times y_0$
goto l_T
 l_6 :

$$p_1(x'_0, y'_0) \leftarrow p_1(x_0, y_0) \wedge \phi$$



Block-based memory model: a pointer is a pair $\langle ref, o \rangle$ where ref uniquely defines a memory object and o defines the byte in the object being point to

$$Env : \mathbb{V} \rightarrow Ptr \quad Ptr = Ref \times Int \quad Mem : Ptr \rightarrow Ptr$$

Concrete memory model:

each allocation (e.g. **malloc**) creates a fresh new object
the number of objects is **infinite**

Abstract memory model:

the number of allocation regions is **finite**
allocation site used as an object reference

Use a whole-program pointer analysis to compute an abstract points-to graph



From Pointer Analysis to CHCs

Run a pointer analysis to disambiguate memory

Produce a side-effect-free encoding by:

replacing each memory object o to a logical array A_o

replacing memory accesses to a pointer p within object o to array reads and writes over A_o

$$v := *(&p + i) \mapsto v = \text{read}(A_o, i)$$
$$*(&p + i) := v \mapsto A'_o = \text{write}(A_o, i, v)$$

each write on A_o produces a new version of A'_o representing the array after the execution of the memory write

Accuracy of pointer analysis is vital for CHC solver's scalability:

resolve aliasing at encoding time

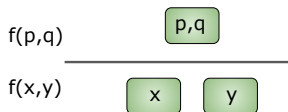


CHCs Using a Context-Insensitive Pointer Analysis

```
void f(int* x, int* y) {
    *x = 1;
    *y = 2;
}

void g(int* p, int* q,
      int* r, int* s) {
    f(p, q);
    f(r, s);
}
```

Assume p and q may alias



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}
```

Assume p and q may alias

f(p,q)

x,y,p,q

f(x,y)

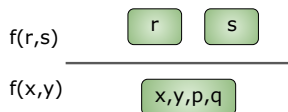


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f(r,s)

f(x,y)

x,y,p,q,r,s



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```

$$S_f(x, y, a_{xy}, a''_{xy}) \leftarrow \\ a'_{xy} = \text{write}(a_{xy}, x, 1) \wedge \\ a''_{xy} = \text{write}(a'_{xy}, y, 2)$$

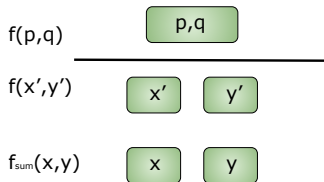
$$S_g(p, q, r, s, a_{pqrs}, a''_{pqrs}) \leftarrow \\ S_f(p, q, a_{pqrs}, a'_{pqrs}) \wedge \\ S_f(r, s, a'_{pqrs}, a''_{pqrs})$$



Sound CHCs Using a Context-Sensitive Pointer Analysis

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Assume p and q may alias

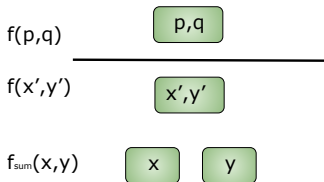


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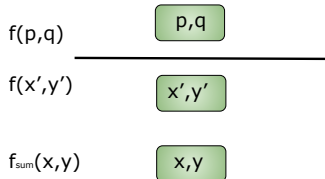
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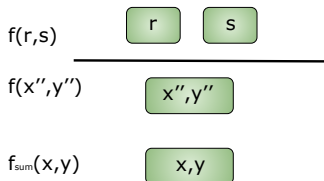


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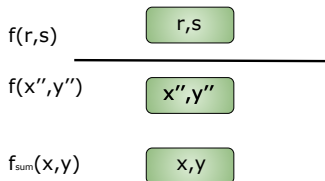


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$$S_g(p, q, r, s, a_{pq}, a_{rs}, a'_{pq}, a'_{rs}) \leftarrow$$
$$S_f(p, q, a_{pq}, a'_{pq}) \wedge$$
$$S_f(r, s, a_{rs}, a'_{rs})$$

Good compromise:

context-sensitive: calls to f do not merge $\{p,q\}$ and $\{r,s\}$
ensure CHCs are sound



A Context-Sensitive Memory Model for Verification of C/C++ Programs*

Arie Gurfinkel¹ and Jorge A. Navas²

it is unification-based (as LLVM-DSA)

it is context-, field-, and array-sensitive

it covers a relevant subset of C/C++ programs that supports:

- dynamic memory allocation

- type unions, pointer arithmetic, pointer casts

- inheritance, function/method calls, etc

it significantly boosts CHC solvers

<https://github.com/seahorn/sea-dsa>



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Main solving engine in SeaHorn

now the default (and only) CHC solver in Z3

<https://github.com/Z3Prover/z3>

dev branch: <https://github.com/agurfinkel/z3>

Supported SMT-theories:

LIA and LRA

quantifier-free theory of arrays

universally quantified theory of arrays + arithmetic

best-effort support for bit-vectors, non-linear arithmetic, etc

Support for non-linear CHCs:

for procedure summaries in inter-procedural verification conditions

for compositional reasoning: assume-guarantee, thread modular, etc.

Based on IC3/PDR-based model checking



Abstract Domains

numerical domains: intervals, zones, boxes, etc

3rd party libraries: apron and elina

arrays and symbolic domains

Analysis of a language-independent core with plugin for LLVM

fixpoint engine based on Bourdoncle's WTO

widening/narrowing strategies

Crab-Llvm: translates to Crab language and integrates optimizations/analysis of LLVM bitcode

Support for inter-procedural and backward analysis

Extensible and open C++ library

Publicly available

<https://github.com/seahorn/crab>

<https://github.com/seahorn/crab-llvm>



Numerical domains

intervals + congruences: $5 \leq x \leq 10 \wedge x \bmod 2 = 0$

zones: $x - y \leq k$

wrapped intervals: intervals on machine-arithmetic integers

non-convex:

DisIntervals: $x \leq -1 \vee x \geq 1$

boxes: boolean combination of intervals

Symbolic domains

terms: numerical domains + uninterpreted functions

$x \leq 10 \wedge y = f(\dots) \wedge z = f(\dots) \rightarrow x \leq 10 \wedge y = z$

$b = \text{write}(a, i, x) \wedge y = \text{read}(a, i) \rightarrow x = y$

Array domains

array smashing: one summarized variable per array (weak updates)

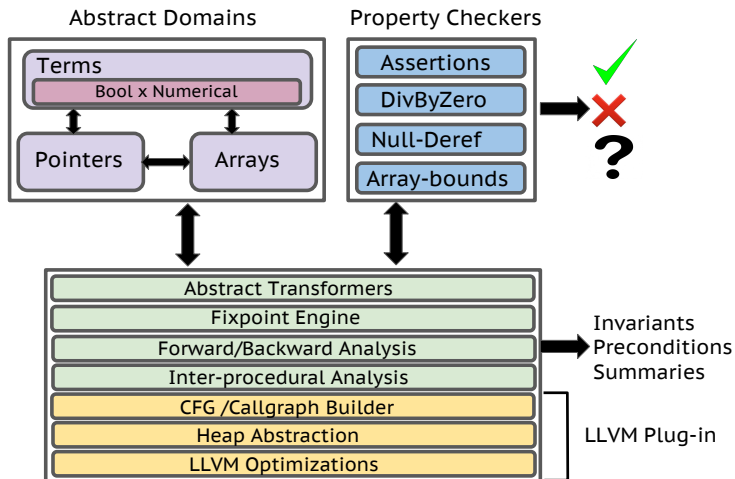
array expansion: one scalar variable per array element (strong updates)

partition-based: weak+strong updates

Apron and Elina: octagons, polyhedra, etc



Crab Architecture and LLVM plug-in



SeaHorn translate CHCs to different formats
SMTLIB2, Boogie, CLP, MCMT, etc

Spacer and Crab generate invariants

Invariant generation is a hard problem

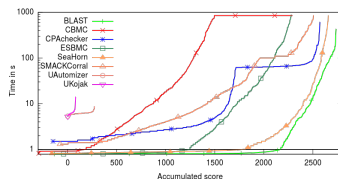
BMC engine for bit-level precision

ML-based learning synthesis engine to complement Spacer
and Crab



SV-COMP

6,000+ files, each 1K-100K LOC



Autopilot code (absence of buffer overflows)



Verify Level 5 requirements of the NASA LADEE software stack:
Manually encode requirements in Simulink models
Verify that the requirements hold in auto-generated C



Build verification technology from scratch is hard

We have built many reusable verification components:

- C/C++ front-ends by reusing compiler technology
- model checking algorithms
- abstract interpretation techniques
- symbolic execution/BMC engines
- pointer analyses

Tested on C device drivers and embedded C/C++ software

Current/future work:

- Making more efficient memory safety checker
- Building executable counterexamples
- Boosting BMC and Spacer with abstract interpretation
- Arrays, machine-arithmetic, FP, new memory models



Thank you!



For latest news, blog posts, publications

`http://seahorn.github.io/`

Open-source software components:

`https://github.com/seahorn/seahorn`

`https://github.com/seahorn/sea-dsa`

`https://github.com/agurfinkel/z3`

`https://github.com/seahorn/crab`

`https://github.com/seahorn/crab-llvm`



- Executable Counterexamples in Software Model Checking. **VSTTE 2018**
- A Context-Sensitive Memory Model for Verification of C/C++. **SAS 2017**
- Synthesizing Ranking Functions from Bits and Pieces. **TACAS 2016**
- Exploiting Sparsity in Difference-Bound Matrices. **SAS 2016**
- An Abstract Domain of Uninterpreted Functions. **VMCAI 2016**
- Finding Inconsistencies in Programs with Loops. **LPAR 2015**
- Compositional Verification of Procedural Programs using Horn Clauses over Integers and Arrays. **FMCAD 2015**
- The SeaHorn Verification Framework. **CAV 2015**
- SMT-Based Model Checking for Recursive Programs. **CAV 2014**



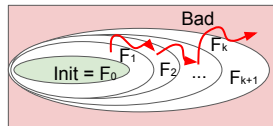
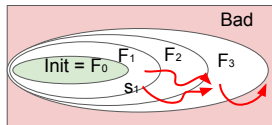
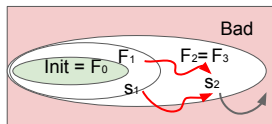
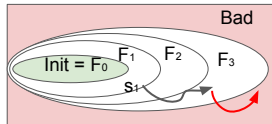
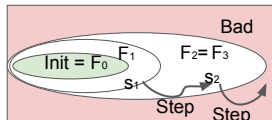
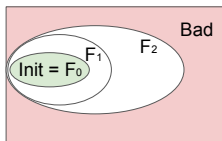
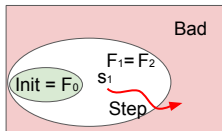
Extra Material



IC3/PDR in One Slide

Invariants

$F_0 = \text{Init}$
 $F_i \Rightarrow F_{i+1}$
 $F_i \text{ and Step} \Rightarrow F_{i+1}$
 $F_i \Rightarrow \text{not Bad}$



Repeat

SAT(F_k and Step and Bad') ?
 SAT(F_{k-1} and Step and s_k') ?

....

$F_{k-1} = F_{k-1}$ and not s_{k-1}
 $F_k = F_k$ and not s_k

if s_k is reachable **then** CEX
else strengthen F_k to exclude s_k

until F_k and Step \Rightarrow not Bad

If $F_k \Rightarrow F_{k-1}$ **then** SAFE
else $k=k+1$



Given F_0, F_1, \dots, F_k , set $F_{k+1} = \neg \text{Bad}$

Apply a backward search:

- 1 Find predecessor s_k in F_k that can reach Bad
check if $F_k \wedge \text{Step} \wedge \text{Bad}'$ is sat
- 2 If none exists, then if $F_{k+1} \Rightarrow F_k$ return "safe". Otherwise, move to next iteration
- 3 If exists, then try to find a predecessor s_{k-1} to s_k in F_{k-1}
check if $F_{k-1} \wedge \text{Step} \wedge s_k'$ is sat
- 4 If none exists, then $F_k = F_k \wedge \neg s_k$ and go back to 3
- 5 Otherwise, recur on (s_{k-1}, F_{k-1})

If we reach *Init* then exists a CEX!



Theories with infinite models:

- cannot block one state at a time

- cannot enumerate all possible predecessors

Non-linear CHCs:

- increase the number of predecessors



Generalize predecessors: $F_{k-1} \wedge \text{Step} \wedge s'_k$

Find a cube m st $m \Rightarrow \exists V'. F_{k-1} \wedge \text{Step} \wedge s'_k$

Block more than one state

$s \models F_k \wedge \text{Step} \wedge \text{Bad}$ and $F_{k-1} \wedge \text{Step} \wedge s$ is unsat

$F_{k-1} \wedge \text{Step} \Rightarrow \neg s$ iff $\neg s \wedge F_{k-1} \wedge \text{Step} \Rightarrow \neg s$

$\neg s$ is **inductive relative** to F_{k-1}

Find c st $c \Rightarrow \neg s$, $c \wedge F_{k-1} \wedge \text{Step} \Rightarrow c$, and $\text{Init} \Rightarrow c$.

If one exists $F_k = F_k \wedge c$

Moreover, for every $i \leq k$ $F_i = F_i \wedge c$ because c is also inductive relative to $F_{k-2}, \dots, F_0!$

Push forward

if $c \in F_k$ and $c \notin F_{k+1}$ and $F_k \wedge c \wedge \text{Step} \Rightarrow c'$ then

$F_{k+1} = F_{k+1} \wedge c$ (for all $1 \leq k \leq N-1$)

